Superluminal tunneling of microwaves in smoothly varying transmission lines

A. B. Shvartsburg
Joint Institute of High Temperatures, Russian Academy of Sciences, Moscow, Russian Federation

M. Marklund, G. Brodin, and L. Stenflo
Department of Physics, Umeå University, SE–901 87 Umeå, Sweden
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Tunneling of microwaves through a smooth barrier in a transmission line is considered. In contrast to standard wave barriers, we study the case where the dielectric permittivity is positive, and the barrier is caused by the inhomogeneous dielectric profile. It is found that reflectionless, superluminal tunneling can take place for waves with a finite spectral width. The consequences of these findings are discussed, and an experimental setup testing our predictions is proposed.

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I. INTRODUCTION

Tunneling is a fundamental process related to the dynamics of various kinds of waves. This phenomenon was already pointed out in Gamow's famous work [1] on nuclear α decay, where the probability of penetration of α particles with energy E through a potential barrier with height U_0 (with $U_0 > E$) was found to be exponentially small, but finite. However, the Gurney and Condon attempt [2] to find the velocity v_t and the transit time τ of such a tunneling revealed a basic problem of the theory; how should one define these quantities in the "classically forbidden zone" ($U_0 > E$), where both the values of v_t and τ are imaginary?

Three decades later the interest in this problem was renewed in Hartmann's paper [3], where the time of tunneling of a particle with energy E through a barrier was determined via the phase of the complex barrier transmission function $T = |T| \exp(i\varphi_t)$, which gives the tunneling time

$$\tau = \hbar \frac{\partial \varphi_t}{\partial E}, \tag{1}$$

where $2\pi\hbar$ is Planck's constant. Using the well-known expressions for the transmission function of a rectangular barrier with width d, Hartmann [3] showed that for the case of a thick barrier ($|\mathbf{p}| d \gg \hbar$, where $\mathbf{p}$ is the particle momentum), the time τ becomes independent of d, and thus for a sufficiently thick barrier the tunneling speed v_t can reach the superluminal velocity $v_t > c$. This conclusion, referred to in the literature as the Hartmann paradox, can be deduced from standard formula given in many textbooks. It has stimulated a hot debate which is still going on [4–7]. However, a direct measurement of the electron tunneling time through a quantum barrier has proved to be an intricate task. The idea to verify Hartmann's conclusion by means of the classical effect of electromagnetic (EM) wave tunneling through macroscopic wave barriers was then proposed. Its basis was the formal similarity between the stationary Schrödinger equation and the usual wave equation, describing both propagating and evanescent EM modes in continuous media.

This idea gave rise to several attempts to examine the tunneling times of EM waves in microwave and optical ranges. Thus, radio wave experiments with tunneling of the TE_{01} mode in an "undersized" metallic waveguide [8], were by some authors interpreted in favor of the concept of superluminal phase times for tunneling EM waves. These conclusions were reached by means of the expression [9]

$$\tau_B = \sqrt{\left(\frac{\partial \varphi_t}{\partial \omega}\right)^2 + \left(\frac{\partial \ln|T|}{\partial \omega}\right)^2} \tag{2}$$

generalizing Eq. (1) to wave tunneling through a barrier characterized by a complex transmission coefficient. However, measurements of the group delay time for the fundamental TE_{01} mode near the cutoff frequency had shown that each of the quantities τ (1) and τ_B (2) correspond to experiments only at some restricted frequency ranges. Tunneling through such homogeneous opaque barriers is accompanied by strong reflections, attenuation and reshaping of the transmitted signal. The destructive interference between the incident and reflected parts of the pulse was considered in Ref. [10]. as a mechanism for suppressing the tail of the transmitted pulse, causing a superluminal motion of the pulse peak. Similar problems in media with a negative dielectric premittivity (e.g., a plasma) or metamaterials (i.e., both negative dielectric premittivity and magnetic permeability, leading to a negative refractive index) were considered in Refs. [11,12], with a focus on the effects due to broadband pulses.

Another approach to this problem is based on the use of a heterogeneous photonic barrier, characterized by a curvilinear profile of the dielectric susceptibility $\epsilon(z)$ across the barrier [13]. Propagation of waves in this geometry is characterized by the following inhomogeneity induced phenomena. (a) The appearance of an easily controlled cutoff frequency Ω, depending on the gradient and curvature of the profile $\epsilon(z)$. (b) Formation of a tunneling regime for waves with frequency $\omega < \Omega$ in dielectric media with $\epsilon > 0$ and $d\epsilon/dz < 0$. (c) Reflectionless (nonattenuative) tunneling providing 100% transfer of wave energy through a barrier for some frequency $\omega_c < \Omega$. These properties were examined in Refs. [13,14] for thin dielectric nanofilms, characterized by a smooth spatial variation of $\epsilon(z)$ at the subwavelength nanoscale. Access to such films, being available due to recent progress in nanotechnology [15], remains, however, a challenging technological problem.

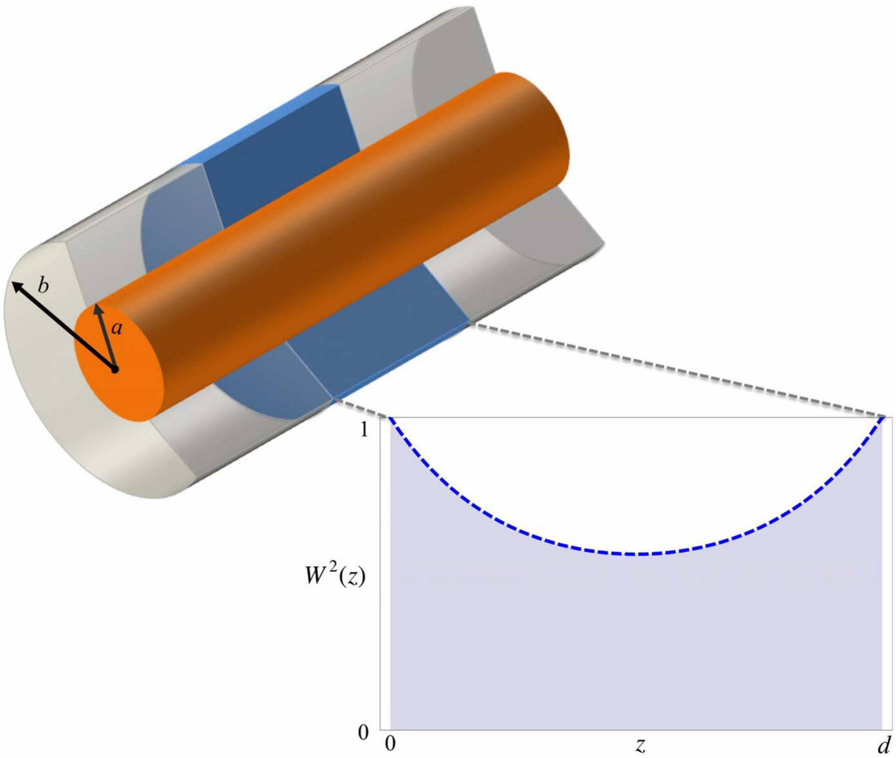

FIG. 1. (Color online) A schematic picture of a coaxial waveguide with a gradient wave barrier in segment 1 between the empty segments 0 and 2. A spatial distribution of the normalized impedance $W^2(z)$ plotted as a function of z ($y^2=1/3$) can also be seen.

The purpose of the present paper is to unite the advantages of both microwave and optical tunneling systems, to enable measurements of substantial phase shifts of GHz microwaves in the regime of nonattenuative tunneling through heterogeneous dielectric layers, characterized, unlike the optical system, by cm scales for the inhomogeneity. A candidate for such a device is the coaxial transmission line (TL), using the fundamental TEM mode. This TL is chosen due to the following salient features, resembling the properties of EM waves in free space [16]. (a) The velocity of the TEM mode is known to coincide with the free space light velocity c. (b) The polarization structure of the TEM mode contains only the transverse field components of **E** and **H**, and not any longitudinal component. (c) The TEM mode does not possess any frequency dispersion.

The organization of the present paper is as follows. Tunneling of the TEM mode through a segment of a coaxial TL, containing a diaphragm made from a gradient dielectric metamaterial is considered in Sec. II. Conditions for reflectionless tunneling of a TEM wave through this diaphragm, accompanied by a substantial phase shift, are examined in Sec. III. The parameters of an interferometric scheme for measurements of this phase shift are presented in Sec. IV. Finally, a brief discussion of the results is given in the Conclusion (Sec. V).

II. EXACTLY SOLVABLE MODEL FOR AN INHOMOGENEOUS TRANSMISSION LINE

Let us consider a coaxial waveguide permitting EM wave propagation between two infinitely conducting cylinders with radii b and a ($b>a$) shown in Fig 1. The fundamental TEM mode is assumed to propagate along this waveguide (z direction). The regions $z\leq 0$ (region 0) and $z\geq d$ (region 2) are assumed to be vacuum, whereas region 1 ($0\leq z\leq d$) is filled with an inhomogeneous dielectric layer, fabricated from a metamaterial, where the dielectric susceptibility is varying such that

$$\epsilon_1 = \epsilon_m W^2(z), \tag{3}$$

where $W^2(z)$ [with $W(0)=1$ and $W(d)=1$] is a dimensionless positive function and ϵ_m is the maximum value of ϵ reached at the ends $z=0$ and $z=d$ of the inhomogeneous region. The spatial variations of the current I and the voltage V along this transmission line are governed by the well-known equations [16]

$$\frac{\partial I}{\partial z} + C\frac{\partial V}{\partial t} = 0 \quad \text{and} \quad \frac{\partial V}{\partial z} + M\frac{\partial I}{\partial t} = 0. \tag{4}$$

Here M and C are the inductance and capacitance per unit length. Losses are ignored. The values C_n and M_n for each region are

$$C_n = \frac{2\pi\epsilon_0\epsilon_n}{\ln\left(\dfrac{b}{a}\right)} \quad \text{and} \quad M_n = \frac{\mu_0\mu_n}{2\pi}\ln\left(\frac{b}{a}\right), \tag{5}$$

where $n=0,1,2$ and where ϵ_0 and μ_0 are the dielectric susceptibility and magnetic permeability of vacuum, respectively. To solve the system (4) in the segment $n=1$, we introduce the generating function ψ defined by

$$V = -M_1\frac{\partial \psi}{\partial t} \quad \text{and} \quad I = \frac{\partial \psi}{\partial z}. \tag{6}$$

Next, considering a harmonic dependence $\propto\exp(-i\omega t)$ we obtain

$$\frac{\partial^2 \psi}{\partial z^2} + \frac{W_n^2(z)}{v_n^2}\omega^2 \psi = 0, \tag{7}$$

where $v_n^2 = 1/M_n C_n$ such that $v_0 = v_2 = c$ and $W_0 = W_2 = 1$. Let us now consider a model where $W_1^2(z)$ contains two free parameters L_1 and L_2 which can be considered as the characteristic spatial scales of the inhomogeneity

$$W_1^2 = \frac{1}{\left(1 + \frac{z}{L_1} - \frac{z^2}{L_2^2}\right)^2}. \tag{8}$$

These parameters can be expressed in terms of the minimum value of the dielectric susceptibility $\epsilon_{\min}$ reached in region 1 due to the profile (8). Introducing the ratio $y = L_2/2L_1$ we find [17]

$$y = \sqrt{\frac{\epsilon_m^{1/2}}{\epsilon_{\min}^{1/2}} - 1}, \quad L_2 = \frac{d}{2y} \quad \text{and} \quad L_1 = \frac{d}{4y^2}. \tag{9}$$

Next we introduce the phase path length $\eta(z) = \int_0^z [W(z_1) dz_1]$, and the new function $\psi = F/\sqrt{W}$. This leads to the resulting equation

$$\frac{\partial^2 F}{\partial \eta^2} - q^2 F = 0 \tag{10}$$

with $q^2 = (\omega^2/v_1^2)(u^2 - 1)$ and $u = \Omega/\omega$ where $u > 1$. Here Ω is a cutoff frequency arising due to the inhomogeneous profile $W_1^2(z)$. It is

$$\Omega = \frac{2yv_1(1+y^2)^{1/2}}{d} \quad \text{where} \quad v_1 = \frac{c}{\sqrt{\epsilon_m}}. \tag{11}$$

Unlike the evanescent modes in Lorentz media [18], characterized by a local frequency dispersion of natural media, Eq. (11) shows the possibility of tunneling of LF waves ($\omega < \Omega$) in a metamaterial with nonlocal dispersion. The solution for F is a sum of a forward and a backward propagating wave, i.e., $F = e^- + Qe^+$ such that

$$\psi = \frac{A(e^- + Qe^+)}{\sqrt{W}}, \tag{12}$$

where $e^- = \exp(-q\eta)$ and $e^+ = \exp(q\eta)$ and A is a normalization constant. Using Eqs. (6) the voltage and current are

$$V_1 = \frac{i\omega M}{v_1} A \left[\frac{e^- + Qe^+}{\sqrt{W}}\right]$$

and

$$I_1 = Aq\sqrt{W}\left[\frac{1}{qL_2}\left(y - \frac{z}{L_1}\right)(e^- + Qe^+) - (e^- - Qe^+)\right], \tag{13}$$

respectively. The continuity of the current and voltage at $z = 0$ gives us the reflection coefficient

$$R_v = \frac{1+\chi}{1-\chi} \tag{14}$$

with

$$\chi = \frac{iZ_0 N}{Z_1}\left[\frac{1}{2qL_1} - \frac{1-Q}{1+Q}\right], \tag{15}$$

where $N = \sqrt{u^2 - 1}$ whereas Z_0 and Z_1 are the impedances of regions 0, 2, and 1, respectively, i.e.,

$$Z_0 = Z_2 = \sqrt{\frac{M_0}{C_0}} \quad \text{and} \quad Z_1 = \frac{Z_0}{\sqrt{\epsilon_m}}. \tag{16}$$

The boundary condition at $z = d$ gives

$$Q = \frac{\exp(2q\eta_0)\left[\beta N + \frac{\gamma}{2} + i\right]}{\left[\beta N - \frac{\gamma}{2} - i\right]}, \tag{17}$$

where

$$\beta = \frac{Z_0}{Z_1} = \sqrt{\epsilon_m}, \quad \gamma = \frac{N\beta}{\omega L_1} = \frac{2\beta u y}{\sqrt{1+y^2}},$$

$$\eta_0 = \eta_0(d) = L_2/(1+y^2)^{1/2}, \quad l_0 = \ln(y_+/y_-)$$

and

$$y_\pm = (1+y^2)^{1/2} \pm y. \tag{18}$$

Thus, we have found a spatial structure of the E and H components of the evanescent TEM mode. Unlike the traditional concept of a homogeneous coaxial TL, which is known to have no cutoff frequency for the fundamental TEM mode [16], our inhomogeneous region for a coaxial waveguide has been shown to provide a cutoff frequency for this mode without any deformation of the coaxial cross section. The drastic consequences of this inhomogeneity induced effect for reflectance and transmission of the TEM mode are discussed below.

III. WINDOWS OF TRANSPARENCY FOR THE EVANESCENT WAVE (REFLECTIONLESS TUNNELING OF THE TEM MODE)

In order to find the reflectance of the gradient wave barrier, we substitute the formula (17) for Q, into Eqs. (14) and (15). This yields the explicit expression for the complex reflection coefficient

$$R_v = \frac{\tanh(q\eta_0)\left(1+\frac{\gamma^2}{4}+\beta^2N^2\right)-\gamma\beta N}{\tanh(q\eta_0)\left(1-\frac{\gamma^2}{4}-\beta^2N^2\right)+\gamma\beta N+i[2\beta N-\gamma\tanh(q\eta_0)]}. \tag{19}$$

Substitution of Eq. (19) into the continuity condition for the voltage at $z=0$: $V_{\rm in}(1+R_v)=i\omega_c^{-1}A(1+Q)$, where $V_{\rm in}$ is the voltage of the incident wave, determines the normalization constant A. Then, substitution of the constant A into Eq. (13) gives the complex transmission function $T=|T|\exp(i\varphi_t)$, i.e.,

$$|T| = \sqrt{1-|R_v|^2} \tag{20}$$

with

$$\varphi_t = \tan^{-1}\left\{\frac{\tanh(q\eta_0)\left(1-\frac{\gamma^2}{4}-\beta^2N^2\right)+\gamma\beta N}{2\beta N-\gamma\tanh(q\eta_0)}\right\}. \tag{21}$$

The formula (19) gives the reflectance of a single inhomogeneous layer. If the wave is tunneling through m $(m\geq 1)$ neighboring and identical layers, the total reflection coefficient R_v and phase φ_t are found from Eqs. (19) and (21) by the replacement

$$\tanh(q\eta_0)\to\tanh(mq\eta_0). \tag{22}$$

We note that the reflection coefficient R_v can be zero, which, for a series of m layers, corresponds to the condition

$$\tanh(mq\eta_0) = \frac{\gamma N\sqrt{\epsilon_m}}{\left(1+\frac{\gamma^2}{4}+\epsilon_m N^2\right)}. \tag{23}$$

The parameters γ and η_0 are defined by Eq. (18). Equation (23) determines the normalized critical frequency $u_c=\Omega/\omega_c$ that results in reflectionless tunneling.

To optimize the parameter values for an inhomogeneous reflectionless barrier, one has to choose the maximum and minimum values of the dielectric susceptibility ϵ_m and $\epsilon_{\rm min}$, respectively, and the number of barriers m. The next steps are as follows. (1) Calculation of the values y from Eq. (9) and v_1 from Eq. (11). (2) Solution of Eq. (23) for the unknown normalized critical frequency $u_c=\Omega/\omega_c$ corresponding to the above values for y and v_1. (3) Determination of the product $\omega_c d$ from the condition following from Eq. (11)

$$\omega_c d = \frac{2yv_1(1+y^2)^{1/2}}{u_c} = K. \tag{24}$$

As a numerical example of such reflectionless tunneling one can consider the values $y^2=1/3$, $\epsilon_m=4.912$, $m=2$; in this case the solution of Eq. (24) is $u_c=1.0205$ and the constant $K=1.761010$ cm/s. Then for a frequency $\omega_c/2\pi=1$ GHz ($\lambda=30$ cm) we obtain $d=2.8$ cm$\ll\lambda$. We note that the condition for reflectionless tunneling (complete transmission $R_v=0$, $|T|=1$) can be fulfilled for different frequencies ω and lengths d, linked by Eq. (23). An example of the transmission spectrum for reflectionless tunneling for the normalized frequencies u is presented in Fig. 2(a). The transmission is almost complete, $|T|>0.9$ in a finite spectral range ($1.02<u<1.25$), forming a transparency window for the tunneling mode.

IV. PHASE EFFECTS IN THE TRANSMITTED MODE: SUPERLUMINAL TUNNELING?

The phase shift φ_t of the tunneling mode (21) is depicted in Fig. 2(b), as a function of a normalized frequency u, for fixed parameters ϵ_m and $\epsilon_{\rm min}$. For the reflectionless tunneling under discussion this shift is $\varphi_t=1.6$ rad. This graph can be used for different frequencies ω and lengths d, linked by a constant product ωd. We notice that this constant product ωd determines the phase shift $\varphi_0=\omega d/c$ of a wave with frequency ω accumulated along the path d in free space.

To measure the phase shift φ_t of the tunneling wave, and comparing it with $\varphi_0=\omega d/c$, one can use the interferometer-like scheme depicted in Fig. 3. The TEM mode, produced by

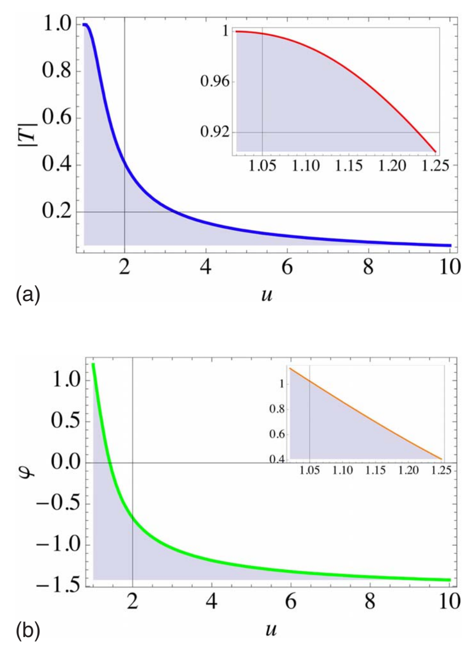

FIG. 2. (Color online) The transmittance spectra for the TEM mode tunneling through two wave barriers in the coaxial waveguide ($\epsilon_m=4.912$, $\epsilon_{\rm min}=3.69$, $m=2$, $y_2=1/3$). $|T|$ is the transparency, u is the normalized frequency, and φ is the phase of the tunneling wave. (a) The spectral window of transparency in the range $1<u<10$ and $1.02<u<1.25$. (b) The phase spectrum of the tunneling mode in the range $1<u<10$ and $1.02<u<1.25$.

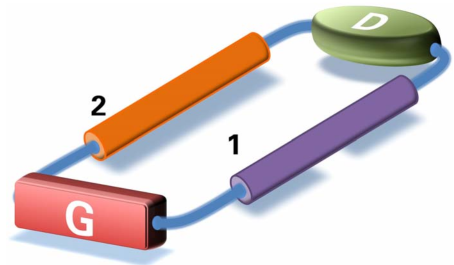

FIG. 3. (Color online) An interferometerlike scheme for measurement of phase shift of the tunneling mode. G is the generator of the TEM mode, 1 is an arm containing two adjoining gradient impedance layers, 2 is an arm containing the empty coaxial, D is a phase detecting element.

a generator G, is splitted between two similar coaxial transmission lines. The space between the cylinders in TL1 is empty, whereas the second one contains the inhomogeneous barrier that has been discussed above. After passage through these arms, the two waves are interfering, and the relative phase shift $\Delta\varphi=\varphi_t-\varphi_0$ can be measured. Let us write

$$\varphi_t=\varphi_0\left(1+\frac{\Delta\varphi}{\varphi_0}\right) \tag{25}$$

and introduce the phase-time delay $\mathcal{T}$ and the relevant velocity v_t, linked by the condition $v_t\mathcal{T}=D=t_0c$. Then, putting $\varphi_0=\omega t_0$ and $\varphi_t=\omega t_0 v_t/c$, we find

$$\frac{v_t}{c}=1+\frac{\Delta\varphi}{\varphi_0},\quad \mathcal{T}=\frac{t_0}{1+\frac{\Delta\varphi}{\varphi_0}},\quad \text{and}\quad t_0=\frac{d}{c}. \tag{26}$$

Thus in the case $\Delta\varphi>0$ ($\Delta\varphi<0$) the scheme discussed will result either in a subluminal ($v_t<c$, $\mathcal{T}>t_0$) or a superluminal ($v_t>c$, $\mathcal{T}<t_0$) regime. For the figures related to the above mentioned reflectionless tunneling ($m=2$, $D=2d$) one can find from Eq. (21), $\Delta\varphi=0.43$ rad, which indicates a superluminal propagation of the evanescent mode with $v_t=1.35c$ and $\mathcal{T}=0.74t_0$. Unlike the phase velocity of propagating wave $v_\phi=\omega/k$, characterizing the continuous accumulation of phase (with the wave number k real), we consider the velocity connected with the phase of evanescent wave, which is not associated with its wave number. Some salient features of such a superluminal tunneling are as follows. (1) Unlike the case with the Hartmann geometry [3], no thick barriers and phase saturation are needed in the present case. (2) The power flow of the TEM mode is completely transmitted by means of the evanescent wave. (3) The energy density of the evanescent mode inside the barrier $(\epsilon|E|^2+\mu|H|^2)/2$ remains positive in each cross section of the barrier.

It is interesting to evaluate the group time delay for this type of tunneling by means of Eqs. (1) and (2). Since we consider the case of reflectionless tunneling ($|T|=1$), the second term in Eq. (2) vanishes, and we can use Eq. (1). Presenting the derivative $\partial\varphi_t/\partial\omega$ from Eq. (21) we can write

$$\tau=\frac{d}{v(u)}\quad\text{and}\quad v(u)=-\frac{K}{u\frac{\partial\varphi_t}{\partial u}}. \tag{27}$$

The constant K is given in Eq. (24). Thus, the time τ is proportional to the length d when the normalized frequency u is given. For the example discussed, i.e., $\omega=1$ GHz and $d=2.8$ cm, the group delay is negative, or $\tau=-0.6$ ns. Here $|\tau|$ is subluminal, i.e., $|\tau|>D/c=0.185$ ns. By considering the frequency $\omega=4$ GHz, ($d=0.70$ cm) keeping the product ωdconstant one finds $\tau=-015$ ns, which implies a superluminal value $|\tau|<D/c$. Note that negative delay times has also been found in plasma media [11], even for broadband wave packets. One has to emphasize that these tunneling wave phenomena do not violate the Einsteinian causality related to traveling waves.

V. CONCLUSIONS

In conclusion, we stress the following points. (1) The possibility of reflectionless tunneling of a microwave TEM mode in a coaxial waveguide with a gradient profile (i.e., gradient media) given by $\epsilon(z)$ has here been examined in the framework of an exactly solvable model. This model describes the effects of heterogeneity-induced dispersion and the controlled formation of a cutoff frequency Ω in the wanted spectral range. Unlike evanescent waves in Lorentz media with a natural local dispersion [18], the tunneling of LF modes ($\omega<\Omega$) arises in gradient media due to nonlocal dispersion. (2) In contrast to the traditional concept of tunneling in media with $\epsilon<0$, another mechanism of tunneling, with $\epsilon>0$ but $\nabla\epsilon<0$, has been considered here. The possibility of a superluminal phase shift, arising in subwavelength wave barriers (beyond Hartmann's condition $kd\gg1$), is demonstrated. An experimental setup, illustrating such tunneling, is suggested. (3) The results, obtained here for microwaves, can easily be generalized to other types of EM waves, keeping the condition of a constant product ωd, with the other parameters of tunneling media being the same. Moreover, the tunneling effects discussed here seem to be rather general, occurring for different types of waves satisfying heterogeneous wave equations for media with continuous spatial variations of its parameters.

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